

Upper Limit on the Cosmic Gamma-Ray Burst Rate from High Energy Diffuse Neutrino Background

Pijushpani Bhattacharjee,^{*} Sovan Chakraborty,[†] Srirupa Das Gupta,[‡] and Kamales Kar[§]

*Theory Division, Saha Institute of Nuclear Physics,
1/AF Bidhannagar, Kolkata 700064. India*

Abstract

We derive upper limits on the ratio $f_{\text{GRB/CCSN}}(z) \equiv R_{\text{GRB}}(z)/R_{\text{CCSN}}(z) \equiv f_{\text{GRB/CCSN}}(0)(1+z)^\alpha$, the ratio of the rate, R_{GRB} , of long-duration Gamma Ray Bursts (GRBs) to the rate, R_{CCSN} , of core-collapse supernovae (CCSNe) in the Universe (z being the cosmological redshift and $\alpha \geq 0$), by using the upper limit on the diffuse TeV–PeV neutrino background given by the AMANDA-II experiment in the South Pole, under the assumption that GRBs are sources of TeV–PeV neutrinos produced from decay of charged pions produced in $p\gamma$ interaction of protons accelerated to ultrahigh energies at internal shocks within GRB jets. For the assumed “concordance model” of cosmic star formation rate, R_{SF} , with $R_{\text{CCSN}}(z) \propto R_{\text{SF}}(z)$, our conservative upper limits are $f_{\text{GRB/CCSN}}(0) \leq 5.0 \times 10^{-3}$ for $\alpha = 0$, and $f_{\text{GRB/CCSN}}(0) \leq 1.1 \times 10^{-3}$ for $\alpha = 2$, for example. These limits are already comparable to (and, for $\alpha \geq 1$, already more restrictive than) the current upper limit on this ratio inferred from other astronomical considerations, thus providing a useful independent probe of and constraint on the CCSN-GRB connection. Non-detection of a diffuse TeV–PeV neutrino background by the up-coming IceCube detector in the South pole after three years of operation, for example, will bring down the upper limit on $f_{\text{GRB/CCSN}}(0)$ to below few $\times 10^{-5}$ level, while a detection will confirm the hypothesis of proton acceleration to ultrahigh energies in GRBs and will potentially also yield the true rate of occurrence of these events in the Universe.

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^{*}Electronic address: pijush.bhattacharjee@saha.ac.in

[†]Electronic address: sovan.chakraborty@saha.ac.in

[‡]Electronic address: srirupa.dasgupta@saha.ac.in

[§]Electronic address: kamales.kar@saha.ac.in

I. INTRODUCTION

Detection of supernova (SN) features in the afterglow spectra of several long duration (typically > 2 s) Gamma Ray Bursts (GRBs) in the past one decade has provided strong support to the hypothesis that a significant fraction, if not all, of the long duration GRBs arise from collapse of massive stars; see, e.g., Refs. [1, 2, 3] for recent reviews. The observed SN features in the GRB afterglow spectra are similar to those usually associated with core-collapse supernovae (CCSNe) of Type Ib/c (see, e.g., [3, 4]). The total energy (corrected for beaming) in keV–MeV gamma rays emitted by typical long-duration GRBs is of order 10^{51} erg, which is roughly the same as the total explosion energy seen in typical CCSNe, although there exists considerable diversity in the energetics of both the SN and the GRB components in the SN-GRB associations observed so far. In particular, the estimated explosion energies of the SNe associated with the GRBs observed so far seem to be somewhat larger than those of normal SNe, leading to this “special” class of SNe being sometimes referred to as “hypernovae”.

The broad class of observational results on SN-GRB associations can be understood within the context of the “collapsar” model [5] in terms of a simple phenomenological picture (see, e.g., [6]) in which the core-collapse of a massive Wolf-Rayet star gives rise to two kinds of outflows emanating from the central regions inside the collapsed star: (a) a narrowly collimated and highly relativistic jet that is responsible for the GRB activity, the jet being driven, for example, by a rapidly rotating and accreting black hole formed at the center in the core-collapse process, and (b) a more wide-angled, quasi-spherical and non-relativistic (or at best sub-relativistic) outflow that goes to blow up the star and gives rise to the supernova. The energies channeled into these two components may in general vary independently, which may explain the diversity of energetics in the observed SN-GRB associations. Actually, depending on the energy contained in it the “GRB jet” may or may not be able to penetrate through the stellar material and emerge outside. Indeed, the fact that the SN-GRB associations observed so far involve CCSNe of Type Ib/c, but not of Type II, may be due to the inability of the GRB-causing jet to penetrate through the relatively larger amount of outer stellar material in the case of Type II SN as compared to that in SNe of Type Ib/c [7]. Considering various factors that may govern the energy channeled into the GRB-causing jet, such as the mass and rotation rate of the black hole, accretion efficiency,

efficiency of conversion of accretion energy into collimated relativistic outflow, and so on, Woosley and Zhang [6] have obtained a rough lower limit of $\sim 10^{48}$ erg/s for the power required for the jet to be able to emerge from the star. This is consistent with the energetics of the GRB components of the SN-GRB associations observed so far.

While SN-GRB associations strongly support the stellar core-collapse origin of most long-duration GRBs, clearly, not all core-collapse events may result in a GRB — the latter depends on whether or not the core-collapse event actually results in a “central engine” (a rotating black hole fed by an accretion disk in the above mentioned phenomenological picture, for example) that is capable of driving the required collimated relativistic outflow. In other words, while every long-duration GRB would be expected to be accompanied by a core-collapse supernova [8], the reverse is not true in general.

What fraction of all stellar core-collapse events in the universe produce GRBs? Methods based on astronomical observations generally indicate the ratio between the cosmic GRB rate and the cosmic Type Ib/c SN rate, $f_{\text{GRB/SNIbc}}$, to be in the range $\sim 10^{-3} - 10^{-2}$ for a wide variety of different assumptions on various relevant parameters such as those that characterize the cosmic star formation rate (SFR), initial mass function (IMF) of stars, masses of Type Ib/c SN progenitors, the luminosity function of GRBs, the beaming factor of GRBs (associated with the fact that individual GRB emissions are highly non-isotropic and confined to narrowly collimated jets covering only a small fraction of the sky), and so on; see, for example, [9, 10] and references therein. The dominant uncertainty in the estimate of $f_{\text{GRB/SNIbc}}$ comes from the uncertainties in the estimates of the local GRB rate and the average GRB beaming factor. However, irrespective of the exact value of the ratio $f_{\text{GRB/SNIbc}}$, it is clear that this ratio is significantly less than unity. This indicates that, apart from just being sufficiently massive stars, the GRB progenitors may need to satisfy additional special conditions. For example, it has been suggested [2] that the degree of rotation of the central iron core of the collapsing star and the metallicity of the progenitor star may play crucial roles in producing a GRB.

In this paper, we discuss an alternative probe of the cosmic GRB rate that uses the predicted high energy (TeV–PeV) diffuse neutrino background produced by GRBs and the experimental upper limit on high energy diffuse neutrino background given by the AMANDA-II experiment in the South Pole [11]. Existence of a high (TeV–PeV) energy diffuse GRB neutrino background (DGRBNuB) due to $p\gamma$ interactions of (ultra)high energy protons ac-

celerated within GRB sources is a generic prediction [12] in most currently popular models of GRBs. This DGRBNuB is subject to being probed by the currently operating and upcoming large volume (kilometer scale) neutrino detectors such as IceCube [13], ANITA [14], ANTARES [15], for example. Since neutrinos, unlike electromagnetic radiation, can travel un-hindered from the furthest cosmological distances, the DGRBNuB automatically includes the contributions from all GRBs in the Universe. Thus, an analysis of the DGRBNuB is likely to provide a good picture of the true rate of occurrence of these events in the Universe. Indeed, as we show in this paper, the upper limits on $f_{\text{GRB/CCSN}}(0)$, the ratio of the local (i.e., redshift $z = 0$) GRB to CCSN rates, derived here from the consideration of DGRBNuB, are, for a wide range of values of the relevant parameters, already more restrictive than the current upper limit on this ratio ($\sim 2.5 \times 10^{-3}$) inferred from other astronomical considerations [9, 10]. Further, non-detection of a diffuse TeV–PeV neutrino background by the up-coming IceCube detector [13] in the South Pole after three years of operation, for example, will imply upper limits on $f_{\text{GRB/CCSN}}(0)$ at the level of few $\times 10^{-5}$, while a detection of the DGRBNuB will provide strong support to the hypothesis of proton acceleration to ultrahigh energies within GRB jets.

Our use of the DGRBNuB in constraining the cosmic GRB rate is in the same spirit as efforts to constrain the cosmic star formation rate (and thereby the cosmic CCSN rate) by using the experimental upper limit (set by the Super-Kamiokande (SK) detector) [16] on the predicted [17] low (few MeV) energy Diffuse Supernova Neutrino Background (DSNuB); see, for example, Refs. [18, 19, 20]. Now that the cosmic SFR including its absolute normalization and thereby the cosmic CCSNe rate have got reasonably well determined by the recent high quality data from a variety of astronomical observations (see, e.g., [20]) (which, by the way, predicts a DSNuB flux that is close to the SK upper limit, implying that the DSNuB is probably close to being detected in the near future), one can begin to think of using this SFR to constrain the ratio of the cosmic GRB rate to CCSNe rate by using the predicted DGRBNuB flux together with the recent upper limits on the diffuse high energy neutrino flux from neutrino telescopes.

We should emphasize here that the upper limits derived in this paper actually refer to the ratio of the rate of GRBs to that of *all* CCSNe including those of Type Ib/c and Type II, although SN-GRB associations observed so far involve SNe of Type Ib/c only. It is known, however, that Type II SNe probably constitute as much as $\sim 75\%$ of all CCSNe;

see, e.g., [21]. Thus, one can get the constraint on the GRB-to-SNIb/c ratio from the GRB-to-CCSNe ratio we obtain here by multiplying the latter by a factor of ~ 4 . Conversely, for later comparison, we shall take the “observed” value of the ratio $f_{\text{GRB/CCSN}}(0)$ to be in the range $2.5 \times (10^{-4} - 10^{-3})$ [9, 10].

Below, we first briefly review the calculation of the DGRBNuB spectrum in section II. The resulting upper limits on $f_{\text{GRB/CCSN}}$ obtained by comparing the DGRBNuB with the current upper limit from AMANDA-II experiment are discussed in section III for various values of some of the relevant GRB parameters. Finally, in section IV we summarize the main results and conclude.

II. DIFFUSE HIGH ENERGY NEUTRINOS FROM GAMMA RAY BURSTS

Starting with the original calculations of Waxman and Bahcall [12], the production of TeV–PeV neutrinos is widely accepted as a generic prediction of the fireball model of GRBs, provided, of course, that protons (in addition to electrons) are accelerated to ultrahigh energies within GRB jets. Reviews of the basic method of calculation of the expected neutrino flux from GRBs can be found, e.g., in [22, 23]. Recent calculations of the GRB neutrino spectra can be found, for example, in [24, 25].

For a given cosmological rate of occurrence of GRBs, the DGRBNuB flux can be calculated by simply convoluting the neutrino production spectrum of individual GRBs with the GRB rate density as a function of redshift, integrating over redshift up to some maximum redshift, and averaging over the intrinsic GRB parameters. In this paper we closely follow the recent calculation of the DGRBNuB spectrum described in Ref. [25] with appropriate modifications for a possible enhanced evolution of the cosmic GRB rate in redshift relative to the cosmic SFR as indicated by a recent analysis of the *Swift* GRB data [26].

In the standard jet fireball model of GRBs (see, e.g., Refs. [22, 27] for reviews), the fundamental source of the observed radiation from GRB is the dissipation of kinetic energy of ultra-relativistic (Lorentz factor $\Gamma \sim \text{few } 100$) bulk flow of matter (caused by ejection from a “central engine”) through formation of shocks which accelerate particles (electrons and protons) to ultra-relativistic energies. The shocks can form either inside the flow material itself due to collision of different shells of matter moving with different Lorentz factors (“internal shocks”) or due to collision of the flow material with an external medium (“external

shocks”). The emission of the observed prompt γ -rays from a GRB source is attributed primarily to synchrotron radiation (with possible additional contribution from Inverse Compton scattering) of high energy electrons accelerated in the internal shocks.

It is expected that along with electrons, protons would also be accelerated at the internal shocks. Since synchrotron energy loss of protons is a slow process, protons can be accelerated to much higher energies than electrons. Indeed, it has been suggested [28] that protons may be accelerated to ultra-high energies in GRB internal shocks and that these UHE protons may explain the observed ultra-high energy (UHE) cosmic rays (UHECR) [29] with energies up to $\sim 10^{20}$ eV. These UHE protons interacting with the photons within the GRB jet would produce high energy charged pions through the photo-pion production process, $p + \gamma \rightarrow n + \pi^+$, and the subsequent decay of each charged pion would give rise to three high energy neutrinos (a ν_μ , a $\bar{\nu}_\mu$ and a ν_e): $\pi^+ \rightarrow \mu^+ + \nu_\mu$, $\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$.

The dominant contribution to photo-pion production comes from the Δ resonance, $p + \gamma \rightarrow \Delta^+ \rightarrow n + \pi^+$, at which the $p\gamma$ interaction cross section peaks with a value $\sigma_{p-\gamma}^{\text{peak}} \approx 5 \times 10^{-28} \text{ cm}^2$. This Δ resonance occurs at the proton threshold energy $\epsilon'_{p,\text{th}}$ (as measured in the GRB wind rest frame — the “comoving frame” hereafter), which satisfies $\epsilon'_{p,\text{th}} \epsilon'_\gamma \approx 0.3 \text{ GeV}^2$, where ϵ'_γ is the comoving frame energy of the colliding photon. In the rest frame of the GRB source (i.e., the central engine), the above threshold condition is $\epsilon_{p,\text{th}} \epsilon_\gamma \approx 0.3 \Gamma^2 \text{ GeV}^2$, where Γ is the bulk Lorentz factor of the GRB wind. In each $p\gamma$ interaction the pion takes away on average a fraction $\sim 20\%$ of the energy of the proton, so each neutrino from the decay of the pion carries $\sim 5\%$ of the energy of the initial proton, assuming that the four final state leptons share the energy of the decaying pion equally. Thus, for a typical photon energy $\epsilon_\gamma \sim 1 \text{ MeV}$ and $\Gamma = 300$, say, we have $\epsilon_{p,\text{th}} \sim 3 \times 10^7 \text{ GeV}$, which will give rise to neutrinos of energy $\sim 1.5 \text{ PeV}$.

The observed prompt γ ray spectra of most GRBs are consistent with photon spectra which are well described by a broken power-law [30, 31]: $dn_\gamma/d\epsilon_\gamma \propto \epsilon_\gamma^{-\beta}$, with $\beta \approx 1.0$ for $\epsilon_\gamma < \epsilon_{\gamma b}$, and $\beta \approx 2.25$ for $\epsilon_\gamma > \epsilon_{\gamma b}$. For typical GRBs, the break energy $\epsilon_{\gamma b} \sim 1 \text{ MeV}$. The normalized photon spectrum in the source rest frame (SRF) can be written as

$$\frac{dn_\gamma}{d\epsilon_\gamma} = 0.2 U_\gamma \epsilon_{\gamma b}^{-1} \begin{cases} \epsilon_\gamma^{-1} & \text{for } \epsilon_\gamma \leq \epsilon_{\gamma b}, \\ \epsilon_{\gamma b}^{1.25} \epsilon_\gamma^{-2.25} & \text{for } \epsilon_\gamma > \epsilon_{\gamma b}, \end{cases} \quad (1)$$

where U_γ is the total photon energy density in the SRF. Note that quantities in the SRF are

related to those in the comoving frame (denoted by primes) by the appropriate powers of the Lorentz Γ factor. Thus, for example, $\epsilon_\gamma = \Gamma \epsilon'_\gamma$, and $U_\gamma = \Gamma^2 U'_\gamma = L_\gamma / 4\pi r_d^2 c$, where L_γ is the photon luminosity in SRF, and $r_d = \Gamma^2 c t_v$ is the characteristic “dissipation” radius where internal shocks are formed and from where most of the radiation is emitted, $t_v \sim (10^{-2} - 10^{-3} \text{ sec})$ being the typical variability timescale of the emitted radiation. Note further that the quantities observed at earth (denoted by the superscript or subscript ‘ob’) are related to those in the SRF through appropriate powers of the redshift factor $(1+z)$. Thus, for example, $\epsilon_\nu^{\text{ob}} = \epsilon_\nu / (1+z)$.

We shall assume that at the internal shock protons are accelerated to a differential spectrum, $dn_p/d\epsilon_p \propto \epsilon_p^{-2}$. The total internal energy in the system, $\mathcal{E}_{\text{total}}$, is assumed to be distributed among electrons, protons and magnetic field as $\mathcal{E}_e = \xi_e \mathcal{E}_{\text{total}}$, $\mathcal{E}_p = \xi_p \mathcal{E}_{\text{total}}$ and $\mathcal{E}_B = \xi_B \mathcal{E}_{\text{total}}$, respectively, with $\xi_e + \xi_p + \xi_B = 1$. We further assume that electrons are efficient radiators, so that $\mathcal{E}_e \approx \mathcal{E}_\gamma = \xi_e \mathcal{E}_{\text{total}}$, where $\mathcal{E}_\gamma = L_\gamma T_d$ is the total isotropic-equivalent energy of the emitted gamma ray photons, T_d being the total duration of the burst.

It is worthwhile noting here that in the fireball model the kinetic energy of the initial bulk flow of matter is predominantly carried by protons, they being ~ 2000 times more massive than electrons. This kinetic energy then is converted into internal energy at the shock, whereby the energy is now shared by protons, electrons and magnetic field. The mechanism by which the energy, which is initially carried mainly in the form of protons, gets transferred to electrons (and magnetic field) is not clear, but the phenomenology of the observed radiation from GRBs requires a significant fraction of the total internal energy to be eventually carried by electrons (see, e.g., [22]). If this energy transfer from protons to electrons is very efficient, it may lead to equipartition of energy between them, i.e., $\xi_p = \xi_e$, but in general one may expect that $\xi_p/\xi_e \geq 1$.

Now, with the proton and photon spectra specified as above, the photo-pion production interactions of the protons with the photons given by the spectrum in eq. (1) can be shown to give rise to the neutrino spectrum [12, 25],

$$\epsilon_\nu^2 \frac{dN_\nu(\epsilon_\nu)}{d\epsilon_\nu} \approx \frac{3}{8} \times 0.56 \times f_\pi(\epsilon_p) \frac{\xi_p}{\xi_e} \mathcal{E}_\gamma \begin{cases} 1 & \text{for } \epsilon_\nu < \epsilon_{\nu*}, \\ (\epsilon_\nu/\epsilon_{\nu*})^{-2} & \text{for } \epsilon_\nu > \epsilon_{\nu*}, \end{cases} \quad (2)$$

where $f_\pi(\epsilon_p)$, the fractional energy loss of a proton to pions during the dynamical expansion time scale of the wind [12], is to be evaluated at $\epsilon_p = 20\epsilon_\nu$. For the photon spectrum given

by equation (1), $f_\pi(\epsilon_p)$ has the form [25]

$$f_\pi(\epsilon_p) = f_0 \begin{cases} 0.88(\epsilon_p/\epsilon_{pb})^{1.25} & \text{for } \epsilon_p < \epsilon_{pb}, \\ 1 & \text{for } \epsilon_p > \epsilon_{pb}, \end{cases} \quad (3)$$

with $f_0 = 0.09 L_{\gamma,51} / (\Gamma_{300}^4 t_{v,-3} \epsilon_{\gamma b, \text{MeV}})$. Here $L_{\gamma,51} = L_\gamma / (10^{51} \text{ erg s}^{-1})$, $t_{v,-3} = t_v / (10^{-3} \text{ s})$, $\Gamma_{300} = \Gamma/300$, and $\epsilon_{\gamma b, \text{MeV}} = \epsilon_{\gamma b} / \text{MeV}$.

In equation (2) the factor $\frac{\xi_p}{\xi_e} \mathcal{E}_\gamma \approx \xi_p \mathcal{E}_{\text{total}} = \mathcal{E}_p$ is just the internal energy contained in protons, of which a fraction f_π goes to pions. The factor $\frac{3}{8}$ comes from the fact that in $p\gamma$ interactions π^+ 's and π^0 's are produced with roughly equal probability and the three neutrinos from the decay of π^+ together carry 3/4-th of the pion's energy. Finally, the factor 0.56 is an overall normalization factor.

The spectrum (2) has two breaks: The first break at $\epsilon_{\nu b} = 0.05 \epsilon_{pb}$ is caused by the break in $f_\pi(\epsilon_p)$ at ϵ_{pb} with

$$\epsilon_{pb} = 1.3 \times 10^7 \Gamma_{300}^2 (\epsilon_{\gamma b, \text{MeV}})^{-1} \text{ GeV}, \quad (4)$$

which, in turn, is due to the break in the photon spectrum (1) at $\epsilon_{\gamma b}$.

The second break is at $\epsilon_{\nu*}$, with [25]

$$\epsilon_{\nu*} = 2.56 \times 10^6 \xi_e^{1/2} \xi_B^{-1/2} L_{\gamma,51}^{-1/2} \Gamma_{300}^4 t_{v,-3} \text{ GeV}, \quad (5)$$

which is due to muon cooling; for neutrino energy above $\epsilon_{\nu*}$ the corresponding energy of the parent muon (coming from the decay of the pion) would be high enough that the characteristic timescale of its energy loss through synchrotron radiation (“cooling”) would be shorter than its decay time scale. Following [25] we shall assume $\xi_e = \xi_B$, in which case $\epsilon_{\nu*}$ becomes independent of these two parameters. For a given \mathcal{E}_γ (which is an observationally measurable quantity), the neutrino spectrum (2) then depends on ξ_p and ξ_e , but only through their ratio, ξ_p/ξ_e , which we shall take to be a free parameter in our calculations below.

With the neutrino spectrum from individual GRBs (in the GRB source rest frame) given by equation (2), the diffuse neutrino flux from all GRBs in the Universe, DGRBNuB, can be calculated as follows:

Let $dn_\nu(\epsilon_\nu^{\text{ob}})$ denote the present number density of neutrinos with energy between ϵ_ν^{ob} and $\epsilon_\nu^{\text{ob}} + d\epsilon_\nu^{\text{ob}}$, which were emitted with energies between ϵ_ν and $\epsilon_\nu + d\epsilon_\nu$ from GRBs at redshifts between z and $z + dz$. Denoting by $R_{\text{GRB}}(z)$ the GRB rate per comoving volume at redshift

z , we can write

$$dn_\nu(\epsilon_\nu^{\text{ob}}) = R_{\text{GRB}}(z)(1+z)^3 \left(\frac{dt}{dz} dz \right) \frac{dN_\nu(\epsilon_\nu)}{d\epsilon_\nu} d\epsilon_\nu (1+z)^{-3}. \quad (6)$$

In this equation the factor $(1+z)^3$ on the right hand side converts the GRB rate per comoving volume to the rate per physical volume while the factor $(1+z)^{-3}$ accounts for the dilution of the number density of the produced neutrinos due to expansion of the Universe.

Using the standard Friedmann relation

$$\frac{dt}{dz} = - \left[H_0(1+z) \sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda} \right]^{-1} \quad (7)$$

(we shall use the standard Λ CDM cosmology parameters, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $H_0 = 70 \text{ km/s/Mpc}$), the total differential flux of neutrinos, $\Phi(\epsilon_\nu^{\text{ob}})$, giving the number of neutrinos (of all flavors) crossing per unit area per unit time per unit energy per unit solid angle, due to all GRBs in the Universe up to a maximum redshift z_{max} can be written as

$$\begin{aligned} \Phi(\epsilon_\nu^{\text{ob}}) &\equiv \frac{c}{4\pi} \frac{dn_\nu(\epsilon_\nu^{\text{ob}})}{d\epsilon_\nu^{\text{ob}}} \\ &= \frac{c}{4\pi} H_0^{-1} \int_0^{z_{\text{max}}} R_{\text{GRB}}(z) \frac{dN_\nu(\epsilon_\nu)}{d\epsilon_\nu} \frac{dz}{\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}}, \end{aligned} \quad (8)$$

where $\frac{dN_\nu(\epsilon_\nu)}{d\epsilon_\nu}$ is given by equation (2) with $\epsilon_\nu = (1+z)\epsilon_\nu^{\text{ob}}$. We assume that, because of the long cosmological baseline, neutrino flavor oscillation distributes the original neutrinos equally into all three flavors.

The source spectrum $\frac{dN_\nu(\epsilon_\nu)}{d\epsilon_\nu}$ for a single GRB is a function of various GRB parameters: L_γ , Γ , T_d , t_v , ξ_p/ξ_e and $\epsilon_{\gamma b}$. We average over the “measurable” GRB parameters L_γ , Γ , T_d and t_v using the procedure described in Ref. [25] using the same distribution functions for these parameters used there [32]. The break energy $\epsilon_{\gamma b}$ can be related to total energy in photons, \mathcal{E}_γ (or equivalently to luminosity L_γ) through the empirical “Amati relation” [33] given by $(\epsilon_{\gamma b}/100 \text{ keV}) = (3.64 \pm 0.04)(\mathcal{E}_\gamma/7.9 \times 10^{52} \text{ erg})^{0.51 \pm 0.01}$. And, as already mentioned, the ratio ξ_p/ξ_e remains as a free parameter.

What remains to be specified is the GRB rate as a function of redshift, $R_{\text{GRB}}(z)$. Stellar core-collapse origin of GRBs as evidenced by CCSN-GRB associations implies that GRB rate should follow CCSN rate, $R_{\text{CCSN}}(z)$, which is proportional to SFR, $R_{\text{SF}}(z)$. Recently, however, an analysis [26] of a reasonably large sample of GRBs with known redshifts from the *Swift* mission [34], together with recent accurate determination of the star formation

history [20], has given strong indication of a possible enhanced evolution of the GRB rate (with redshift) relative to SFR. Taking cue from this we shall allow for a possible effective evolutionary factor in the GRB rate relative to SFR and write

$$R_{\text{GRB}}(z) \propto (1+z)^\alpha R_{\text{SF}}(z), \quad (9)$$

where $\alpha \geq 0$ is a constant, and $R_{\text{SF}}(z)$ (rate per *comoving volume*) is taken as [20, 26]

$$R_{\text{SF}}(z) \propto \begin{cases} (1+z)^{3.44} & \text{for } z < 0.97 \\ (1+z)^{-0.26} & \text{for } 0.97 < z < 4.48 \\ (1+z)^{-7.8} & \text{for } 4.48 < z, \end{cases} \quad (10)$$

with $R_{\text{SF}}(0) = 0.0197 M_\odot \text{ yr}^{-1} \text{ Mpc}^{-3}$. This SFR including its normalization has been derived from and is in concordance with recent accurate data on a variety of different indicators of SFR in the Universe, and is also in conformity with the experimental upper limit on the DSNuB flux given by the Super-Kamiokande experiment [16]. Following the terminology introduced in [19] we shall refer to the above SFR as the “concordance model” of SFR.

The core-collapse supernova rate, $R_{\text{CCSN}}(z)$, is related to $R_{\text{SF}}(z)$ through the Initial Mass Function (IMF), dn/dm , giving the differential mass distribution of stars at formation. Thus,

$$R_{\text{CCSN}}(z) = \frac{\int_{8 M_\odot}^{100 M_\odot} \frac{dn}{dm} dm}{\int_{0.1 M_\odot}^{100 M_\odot} m \frac{dn}{dm} dm} R_{\text{SF}}(z), \quad (11)$$

where, following standard practice, the IMF is assumed to be epoch (redshift) independent (see, e.g., [18, 19]), and we have assumed that all stars more massive than $\sim 8 M_\odot$ undergo core-collapse and die on a time scale short compared to Hubble time. Also, our results are insensitive to the exact value of the upper cut-off of the IMF (chosen to be at $100 M_\odot$ above) as long as it is sufficiently large ($\gtrsim 30 M_\odot$ or so). The SFR (10) assumes an IMF of the form [35], $dn/dm \propto m^{-2.15}$ for $m > 0.5 M_\odot$, and $dn/dm \propto m^{-1.50}$ for $0.1 M_\odot \lesssim m \leq 0.5 M_\odot$. With this, the GRB rate can be written in terms of CCSN rate as

$$R_{\text{GRB}}(z) \equiv f_{\text{GRB/CCSN}}(z) R_{\text{CCSN}}(z) = f_{\text{GRB/CCSN}}(0) (1+z)^\alpha R_{\text{CCSN}}(z), \quad (12)$$

where the the normalized core-collapse event rate in the Universe, $R_{\text{CCSN}}(z)$, using equations (10) and (11), is

$$R_{\text{CCSN}}(z) = 2.60 \times 10^{-4} \text{ yr}^{-1} \text{ Mpc}^{-3} \begin{cases} (1+z)^{3.44} & \text{for } z < 0.97 \\ 12.29 (1+z)^{-0.26} & \text{for } 0.97 < z < 4.48 \\ 4.57 \times 10^6 (1+z)^{-7.8} & \text{for } 4.48 < z. \end{cases} \quad (13)$$

The analysis of Ref. [26] seems to indicate the best-fit value of the evolution index α appearing in equations (9) and (12) to be ~ 1.5 , but in this paper we shall keep α as a free parameter and study the dependence of our derived upper limits on $f_{\text{GRB/CCSN}}(0)$ on α .

III. UPPER LIMITS ON $f_{\text{GRB/CCSN}}$

The DGRBNuB (all flavor) flux calculated from equation (8) together with equations (2) – (5), (12) and (13) with $f_{\text{GRB/CCSN}}(0) = 1$ and $z_{\text{max}} = 6$ (there is negligible contribution from z beyond this value), and averaged over the GRB parameters in the manner described in the previous section, is shown in Figures 1 and 2 (the superscript “ob” has been dropped in these Figures). Figure 1 shows the flux for the equipartition case of $\xi_p/\xi_e = 1$ (i.e., $\xi_p = \xi_e = \xi_B = 1/3$ with our choice of $\xi_e = \xi_B$) for five different values of the GRB rate evolution index α including the case $\alpha = 0$ (no evolution), while Figure 2 shows the flux for different values of the parameter ξ_p/ξ_e with $\alpha = 1.5$, its “best-fit” value from Ref. [26]. In both Figures, we also show the current all flavor 90% C. L. upper limit on $\epsilon_\nu^2 \Phi(\epsilon_\nu)$ from the AMANDA-II experiment [11] [36] and also the projected upper limit from the IceCube experiment after three years of operation [37], both for an assumed spectrum of the form $\Phi(\epsilon_\nu) \propto \epsilon_\nu^{-2}$. The resulting upper limits on $f_{\text{GRB/CCSN}}(0)$ obtained by requiring that $\epsilon_\nu^2 \Phi(\epsilon_\nu)$ not exceed the AMANDA-II limit are shown in Figures 3 and 4 as functions of the parameters α and ξ_p/ξ_e , respectively. For comparison, the range of current estimates of the value of the ratio $f_{\text{GRB/CCSN}}(0)$ derived from various astronomical observations [9, 10] is also indicated in Figures 3 and 4.

It is clear that for a given value of α a higher value of the ratio ξ_p/ξ_e implies a higher predicted level of DGRBNuB flux (see equation (2)), thus giving more stringent constraint on (i.e., a smaller upper-limit value of) $f_{\text{GRB/CCSN}}(0)$. Similarly, for a given value of ξ_p/ξ_e , a higher value of α implies more GRBs at higher redshifts, again implying a higher predicted level of DGRBNuB and consequently more stringent constraint on $f_{\text{GRB/CCSN}}(0)$. Thus, the most conservative limit on $f_{\text{GRB/CCSN}}(0)$ comes from the case $\alpha = 0$ and $\xi_p/\xi_e = 1$. These limits are $f_{\text{GRB/CCSN}}(0) \leq 5.0 \times 10^{-3}$ for $\alpha = 0$, and $f_{\text{GRB/CCSN}}(0) \leq 1.1 \times 10^{-3}$ for $\alpha = 2$. For the “best-fit” value of $\alpha = 1.5$ [26], we have $f_{\text{GRB/CCSN}}(0) \leq 1.7 \times 10^{-3}$ for $\xi_p/\xi_e = 1$. We also see from Figures 3 and 4 that, for a wide range of other values of the parameters ξ_p/ξ_e and α , the upper limits on $f_{\text{GRB/CCSN}}(0)$ derived here from the consideration of high

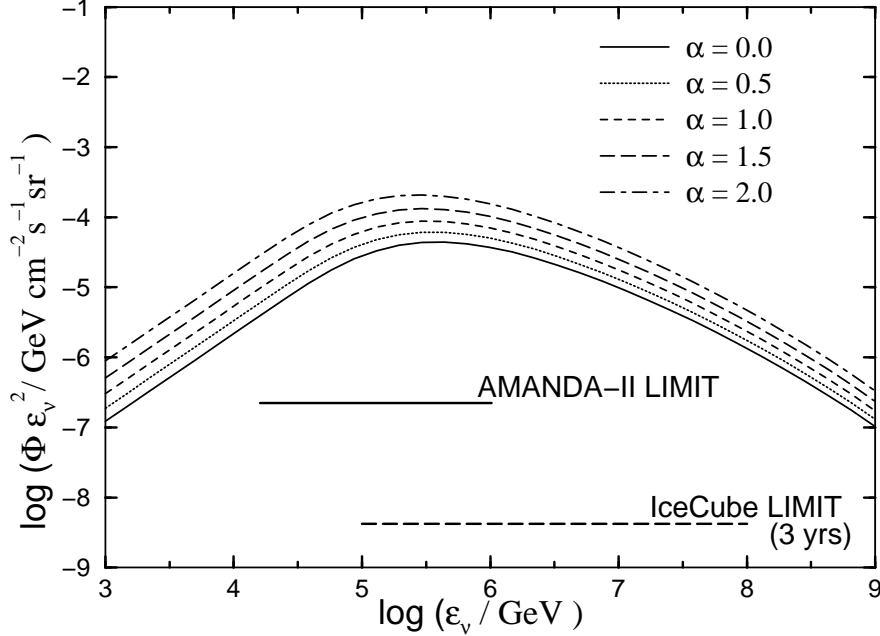


FIG. 1: The total (all flavor) DGRBNuB flux with $f_{\text{GRB/CCSN}}(0) = 1$ for the equipartition case of $\xi_p/\xi_e = 1$ for various values of α , the effective evolution index of the cosmic GRB rate relative to cosmic star formation rate. The current 90% C. L. upper limit on the diffuse neutrino flux given by the AMANDA-II experiment [11] and the projected upper limit from the IceCube experiment after three years of operation [37] are also shown.

energy diffuse neutrino background are already more restrictive than the current upper limit ($\sim 2.5 \times 10^{-3}$) on $f_{\text{GRB/CCSN}}(0)$ inferred from other astronomical considerations [9, 10].

At this point it should be mentioned that the AMANDA-II limit we have used above actually applies specifically to an assumed diffuse neutrino spectrum of the form $\Phi(\epsilon_\nu) \propto \epsilon_\nu^{-2}$. The DGRBNuB spectra shown in Figures 1 and 2 are clearly not of this form. Strictly speaking, therefore, we should calculate the experimental “AMANDA-II” upper limit for our form of the DGRBNuB spectrum and then use that to derive the upper limits on $f_{\text{GRB/CCSN}}(0)$. This can in principle be done by feeding the DGRBNuB spectra calculated above to the detailed detector simulation and optimized signal event selection procedures for the AMANDA experiment. Clearly, this is beyond our scope in this paper. However, use of the ϵ_ν^{-2} AMANDA-II limit in our case here may not be too bad an approximation as a first step since, according to the signal event selection criteria of the AMANDA-II experiment as explained in Ref. [11], it seems reasonable to expect that the dominant contribution to the would-be signal events for our spectrum would come from the region around the broad

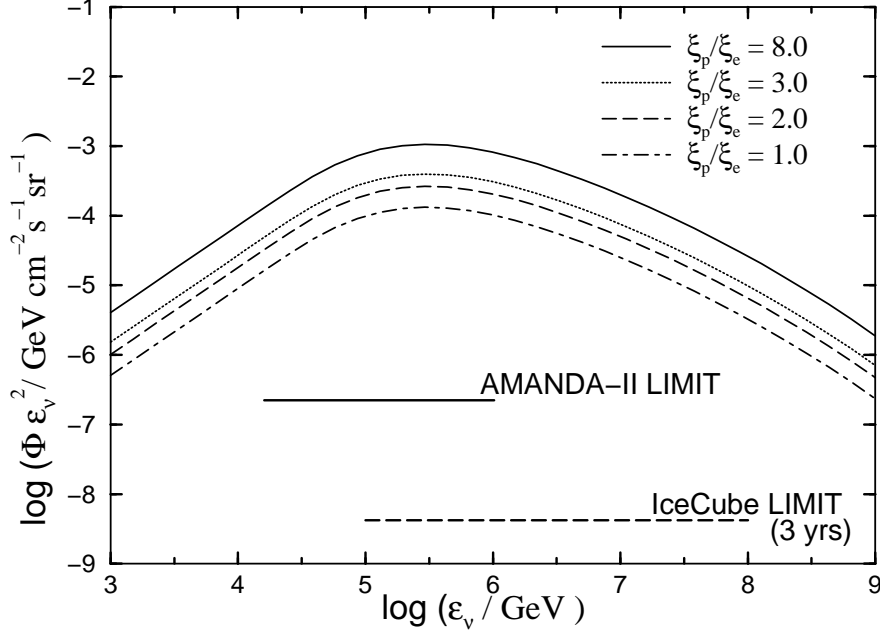


FIG. 2: Same as Fig. 1, but for the fixed value of $\alpha = 1.5$, and various different values of the parameter ξ_p/ξ_e .

peak of the $\epsilon_\nu^2 \Phi$ spectrum where indeed $\Phi \propto \epsilon_\nu^{-2}$, approximately. Thus, while we recognize that the upper limits on $f_{\text{GRB/CCSN}}(0)$ derived here from directly using the ϵ_ν^{-2} AMANDA-II limit in our case should be treated with caution, we do not expect significant changes in our results (by say more than a factor of few) under a more proper evaluation of the experimental “AMANDA-limit” for our spectrum.

It is interesting to note from Figures 3 and 4 that the conservative upper limit on $f_{\text{GRB/CCSN}}(0)$ (obtained with $\xi_p/\xi_e = 1$) for the case of $\alpha = 1.5$, the best-fit value of the evolution parameter [26], is not far above the current estimate of the lower limit on this ratio inferred from other considerations. For larger values of ξ_p/ξ_e the upper limits are even closer to the otherwise estimated lower limit on $f_{\text{GRB/CCSN}}(0)$. This implies that the predicted DGRBNuB flux should be detectable by the upcoming detectors such as IceCube which will have significantly improved sensitivity over that of AMANDA, unless the estimates of $f_{\text{GRB/CCSN}}(0)$ from direct astronomical observations are gross overestimates (which is possible, for example, due to incorrect estimates of the average GRB beaming factor), or that the assumption of proton acceleration to ultrahigh energies within GRB jets is invalid, or both of these.

A caveat in the analysis presented above is that it is based on the standard assumption

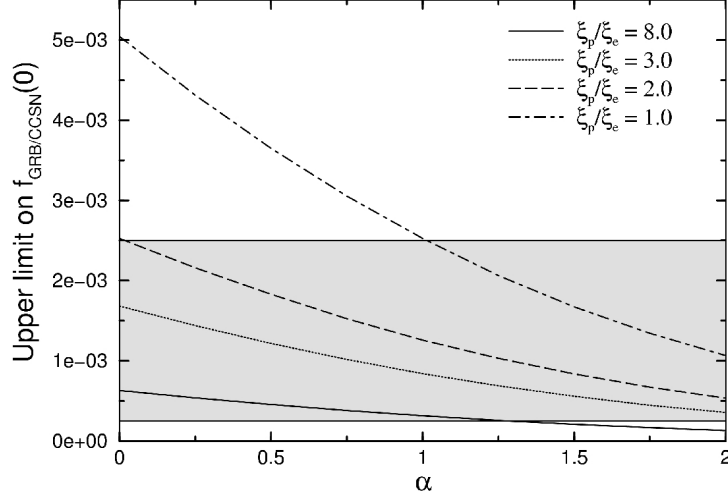


FIG. 3: Upper limits on the fraction $f_{\text{GRB/CCSN}}(0)$, obtained by requiring $\epsilon_\nu^2 \Phi(\epsilon_\nu)$ for the DGRB-NuB to not exceed the AMANDA-II limit, are shown as function of the GRB evolution index α for various values of the parameter ξ_p/ξ_e . The shaded region indicates the range of values of $f_{\text{GRB/CCSN}}(0)$ estimated from other astronomical considerations.

of variability timescales of GRBs on the order of milliseconds, which implies small emission regions and consequently large internal target photon densities for efficient neutrino production through photohadronic processes [38]. While millisecond timescale variability has been seen for many GRBs, this may not always be the case. Efficiency of high energy neutrino production in GRBs in the collapsar model with variability on larger timescales has been studied, for example, in Refs. [24, 39]. Also, neutrino production can be effectively quenched in individual GRBs if Γ , the bulk flow Lorentz factor, is sufficiently large. Clearly, more precise determination of the distribution of the bulk flow Lorentz factor and variability timescale of the GRBs will be useful in calculating the expected level of the diffuse neutrino flux from GRBs more reliably which, together with the results from experiments such as Ice-Cube, should be able to place more precise constraints on the fraction of all stellar collapse events that give rise to GRBs.

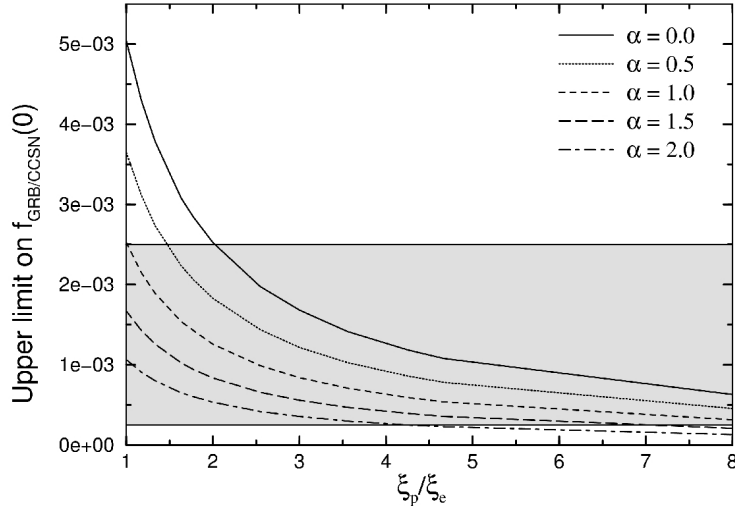


FIG. 4: Same as Fig. 3, but as function of the parameter ξ_p/ξ_e for different values of the GRB evolution index α .

IV. SUMMARY AND CONCLUSIONS

In this paper we have attempted to derive upper limits on the fraction $f_{\text{GRB/CCSN}}$ of all stellar core-collapse events that give rise to GRBs, by using the current experimental upper limit on the high energy (TeV – PeV) diffuse neutrino background given by the AMANDA-II experiment in the South Pole, under the assumption that GRBs are sources of such high energy neutrinos. High energy neutrinos are predicted to be produced within GRB jets through photopion production by protons and subsequent decay of the charged pions, provided protons are accelerated to ultrahigh energies at the internal shocks within GRB jets. In our calculation we have allowed for a possible evolution of the cosmic GRB rate relative to star formation rate. For a wide range of values of various parameters, the upper limits on $f_{\text{GRB/CCSN}}(0)$ derived here from the AMANDA-II results are already more restrictive than the upper limit on this ratio inferred from other astronomical considerations, thus providing a useful independent probe of and constraint on the CCSN-GRB connection. The closeness of the upper limits on $f_{\text{GRB/CCSN}}(0)$ derived here (in particular for the case of enhanced evolution of the GRB rate relative to the star formation rate at high redshifts) to the lower limit on this ratio inferred from various astronomical considerations seems to indicate

that the predicted DGRBNuB flux should be detectable by the upcoming detectors such as IceCube which will have significantly improved sensitivity over that of AMANDA-II. On the other hand, non-detection of the DGRBNuB by the IceCube detector after three years of operation, for example, will give more stringent upper limits on $f_{\text{GRB/CCSN}}$, but at the same time will also imply that either the values of $f_{\text{GRB/CCSN}}$ inferred from direct astronomical observations have been significantly overestimated (which is possible, for example, due to incorrect estimates of the average GRB beaming factor) or that the assumption of proton acceleration to ultrahigh energies within GRB jets is invalid, or both of these. However, more precise determination of the distribution of some of the crucial GRB parameters such as the bulk flow Lorentz factor and variability timescale of the GRBs will be needed to reliably calculate the expected contribution of the GRBs to the high energy diffuse neutrino background, and thereby to determine the upper limits on $f_{\text{GRB/CCSN}}$ more reliably. To conclude, then, the up-coming large volume neutrino telescopes hold immense promise of yielding significant information both on the nature of the fundamental physical process of particle acceleration in GRB sources as well as on the rate of occurrence of these events in the Universe.

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- [1] P. Mészáros, *Ann. Rev. Astron. Astrophys.* **40**, 137 (2002).
 - [2] S.E. Woosley and J. Bloom, *Ann. Rev. Astron. Astrophys.* **44**, 507 (2006).
 - [3] M. Della Valle, in *Gamma-Ray Bursts in the Swift Era*, Sixteenth Maryland Astrophysics Conference, eds. S.S Holt et al, AIP Conference Proceedings, Vol. **836**, p. 367–379 (2006).
 - [4] S.E. Woosley and A. Heger, *Astrophys. J.* **637**, 914 (2006).
 - [5] A. MacFadyen and S. Woosley, *Astrophys. J.* **524**, 262 (1999); W. Zhang, S.E. Woosley and A.I. MacFadyen, *Astrophys. J.* **586**, 356 (2003).
 - [6] S.E. Woosley and W. Zhang, arXiv:astro-ph/0701320v1.
 - [7] SNe of Type Ib (Ic) are thought to arise from progenitors that have lost their hydrogen (and helium) envelop before collapse, whereas the progenitor stars giving rise to SNe of Type II have their hydrogen envelop still in place at the time of collapse.
 - [8] Actually, there may be cases of “failed supernovae”, i.e., the star may fail to explode even

though core-collapse has taken place. We shall ignore these subtleties here and use the term “core-collapse supernova” throughout this paper to denote all core-collapse events.

- [9] D. Guetta and M. Della Valle, arXiv:astro-ph/0612194v1.
- [10] E. Bissaldi et al, arXiv:astro-ph/0702652v1.
- [11] A. Achterberg et al [IceCube Collaboration], arXiv:0705.1315v1 [astro-ph].
- [12] E. Waxman and J. Bahcall, Phys. Rev. Lett. **78**, 2292 (1997); Astrophys. J. **541**, 707 (2000).
- [13] See <http://icecube.wisc.edu/>
- [14] See <http://www.phys.hawaii.edu/~anita/>
- [15] See <http://antares.in2p3.fr/>
- [16] M. Malek et al, Phys. Rev. Lett. **90**, 061101 (2003).
- [17] G.S. Bisnovatyi-Kogan and S.F. Seidov, Ann. N.Y. Acad. Sci. **422**, 319 (1984); L. M. Krauss, S. L. Glashow and D. N. Schramm, Nature **310**, 191 (1984).
- [18] S. Ando and K. Sato, New J. Phys. **6**, 170 (2004) [arXiv:astro-ph/0410061v2].
- [19] L.E. Strigari et al, JCAP **0504**, 017 (2005) [arXiv:astro-ph/0502150v2].
- [20] A.M. Hopkins and J.F. Beacom, Astrophys. J. **651**, 142 (2006).
- [21] E. Cappellaro, M. T. Botticella and L. Greggio, arXiv:0706.1299v1 [astro-ph].
- [22] E. Waxman, in Lect. Notes in Physics, (eds. M. Lemoine and G. Sigl, Springer, Berlin, 2001), Vol. **576**, p. 122. [arXiv:astro-ph/0103186v1].
- [23] F. Halzen and D. Hooper, Rept. Prog. Phys. **65**, 1025 (2002).
- [24] K. Murase and S. Nagataki, Phys. Rev. D **73**, 063002 (2006); K. Murase et al, Astrophys. J. Lett. **651**, L5 (2006).
- [25] N. Gupta and B. Zhang, Astropart. Phys. **27**, 386 (2007).
- [26] M.D. Kistler et al, arXiv:0709.0381v2 [astro-ph].
- [27] T. Piran, Phys. Rep. **333**, 529 (2000).
- [28] E. Waxman, Phys. Rev. Lett. **75**, 386 (1995); M. Vietri, Astrophys. J. **453**, 883 (1995).
- [29] P. Sokolsky and G.B. Thomson, arXiv:0706.1248v1 [astro-ph]; T. Yamamoto (for Auger Collaboration), arXiv:0707.2638v3 [astro-ph];
- [30] D. Band et al, Astrophys. J. **413**, 281 (1993).
- [31] T. Sakamoto et al, Astrophys. J. **629**, 311 (2005).
- [32] The normalization of the log-normal distribution used in the calculations of Ref. [25] to average over each of the parameters Γ , T_d and t_v was incorrect (N. Gupta, private communication).

Here we have used the correctly normalized distributions.

- [33] L. Amati et al, Astron. Astrophys. **390**, 81 (2002); G. Ghirlanda et al, Mon. Not. Roy. Astron. Soc. **361**, 10G (2005).
- [34] N. Gehrels et al, Astrophys. J. **611**, 1005 (2004).
- [35] I.K. Baldry and K. Glazebrook, Astrophys. J. **593**, 258 (2003).
- [36] We have multiplied the single flavor ($\nu_\mu + \bar{\nu}_\mu$) AMANDA-II upper limit given in Ref. [11] by a factor of 3 to get the all-flavor limit assuming the flavor flux ratio of the neutrinos arriving at Earth to be 1:1:1.
- [37] Mathieu Ribordy, et al [IceCube Collaboration], Phys. Atom. Nucl. **69**, 1899 (2006) [arXiv:astro-ph/0509322v1].
- [38] Specifically, we have used, following Ref. [25], a log-normal distribution of variability timescales with a mean at ~ 38 ms and a standard deviation of ~ 30 ms.
- [39] C.D. Dermer and A. Atoyan, Phys. Rev. Lett. **91**, 071102 (2003); New J. Phys. **8**, 122 (2006).